Notre Dame Journal of Formal Logic
Volume XVIII, Number 1, January 1977
NDJFAM

FIRST DEGREE FORMULAS IN CURRY'S LD

ROBERT K. MEYER

In [1], Belnap provided an algebraic semantics for the first-degree fragments of the relevant logics **E** and **R**, i.e., the sets of formulas A such that no implication signs $\rightarrow$ themselves occur within the scope of implication signs. First-degree formulas, also studied from a Kripke-style semantic point of view in Routley's [2], are particularly important because only on such can implication be taken in its natural sense as a *relation* between sentences,[1] which either holds or does not hold, rather than as a *connective* to be applied to sentences to yield further sentences. Arguments against using implication as a connective seem to be losing force as years go by, but both for those who continue to take them seriously and on considerations of general simplicity, independent characterizations of the first-degree fragments of familiar logics are important and interesting. Accordingly, in the present note Belnap's methods will be adapted to give a very simple characterization of the set of valid first-degree formulas in Curry's **D**, as presented in [3].[2] The intuitionist logic **J** comes along to some extent, so it is included in the characterization. And I note here that although I am indebted to Belnap for root insights in the relevant contexts, I am equally indebted to Dunn for his penetrating algebraic analyses and explications of these insights in [6] and [7].

1 I shall take as an underlying language $\mathcal{L}$ one with denumerably many sentential variables, positive connectives &, v, $\supset$, and sentential constants f, F. Formulas A, B, etc., are built up as usual, and the following definitions are entered.

D0. $A \equiv B =_{df} (A \supset B) \,\&\, (B \supset A)$
D1. $\sim A =_{df} A \supset \mathsf{f}$ (**D**-negation)
D2. $\neg A =_{df} A \supset \mathsf{F}$ (intuitionist (**J**) negation)

The following axiom schemes and rule produce a system **DJ**.[3]

A1. $A \supset . B \supset C : \supset : A \supset B . \supset . A \supset C$
A2. $A \supset . B \supset A$
A3. $A \,\&\, B \supset A$

Received October 5, 1971