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A NOTE ON THOMASON'S REPRESENTATION OF S5

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Introduction S. K. Thomason has proved in [3] that a formula is provable in S5 iff all its substitution instances are in H, which is a unique correct set and is Thm($\mathfrak{C}$). In order to prove this, he semantically showed that a formula $A(x_1, \ldots, x_n)$ is valid in S5 (tautology of S5 in the sense of Kripke [2], pp. 11ff.) iff $V^*(A(B_1, \ldots, B_n)) = 1$ for all $B_1, \ldots, B_n$ in $\mathcal{L}_c$ (modal language with proposition constants).

In this paper, we shall show by means other than Kripke's model that $A(x_1, \ldots, x_n)$ is provable in S5 iff $\mu^*(A(B_1, \ldots, B_n)) = 1$ for all classical formulas (without modal symbols), $B_1, \ldots, B_n$, for all μ^*, where μ^* is essentially the same as V^* above, except that μ^* is a valuation for modal formulas with proposition variables. In the last section of this paper, we shall also show a relation between Kripke's partial truth tables and μ^*-valuations.

1 *Formulation of* S5 *and truth valuation* We prepare a countable set of proposition variables, Π, logical connectives, $\vee$, $\sim$, $\Box$, and parentheses, (,). Formulas are defined as usual. For any formulas A and B, we define $A \wedge B$ as $\sim(\sim A \vee \sim B)$, $A \rightarrow B$ as $\sim A \vee B$, $A \leftrightarrow B$ as $(A \rightarrow B) \wedge (B \rightarrow A)$, and $\Diamond A$ as $\sim\Box\sim A$. If A and B are formulas, the following expressions are axioms:

(A1) $(A \vee A) \rightarrow A$.
(A2) $B \rightarrow (A \vee B)$.
(A3) $(A \vee B) \rightarrow (B \vee A)$.
(A4) $(B \rightarrow C) \rightarrow ((A \vee B) \rightarrow (A \vee C))$.
(A5) $\Box A \rightarrow A$.
(A6) $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$.
(A7) $\Diamond A \rightarrow \Box\Diamond A$.

When A and B are formulas, we suppose the following rules of inference:

(R1) If $\vdash A$ and $\vdash A \rightarrow B$, then $\vdash B$.
(R2) If $\vdash A$, then $\vdash \Box A$.

For any formula A, we say that A is a classical formula iff A contains none of $\Box$ and $\Diamond$. $A(x_1, \ldots, x_n)$ denotes a formula, A, having exactly n distinct

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