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A NOTE ON THE COMPLETENESS PROOF FOR NATURAL DEDUCTION

DAVID W. BENNETT

Brief as it is, the argument of my earlier paper[1] can be further simplified to put it easily within reach of beginning students of logic. As in that paper, let a system of natural deduction be based on *negation, conjunction*, and *universal quantification*, with the standard rules of *indirect proof, simplification* and *conjunction*, and *instantiation* and *generalization* governing these three operations, respectively. For easier exposition we also include now another rule, clearly redundant, for simplifying double negations.

Let a deduction D be given, each of whose assumptions is undischarged in D and remains undischarged in any extension of D. Then the following rules for appending steps to D will define a certain extension D′ of D whose assumptions are likewise undischarged and undischargeable. If the first step in D is of a form treated by one of the rules 1-5 below introduce a new formula or formulas as the rule instructs, go on to the second step, and so on until all the formulas of D have been harvested and D′ has been reached. Repetitions may be omitted.

(1) From a step in D of the form $P\&Q$ introduce inferences in D′ of the forms P and Q, by *simplification.*
(2) From a step in D of the form $\forall xFx$ introduce inferences in D′ of the form Fa, by *instantiation,* using each free variable in D and the first free variable not in D. (Free and bound variables are distinguished typographically.)
(3) From a step in D of the form $--P$ introduce an inference in D′ of the form P, by *double negation.*
(4) From a step in D of the form $-(P\&Q)$ introduce an assumption in D′ of form $-P$, or, in case this would be dischargeable, introduce an assumption

1. "An elementary completeness proof for a system of natural deduction," *Notre Dame Journal of Formal Logic*, vol. XIV (1973), pp. 430-432.

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