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A TERNARY UNIVERSAL DECISION ELEMENT

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1 *Introduction* A universal decision element is one which may be used to define any of the functors of one or two arguments by the substitution of variables x, y, etc., or constants in its arguments, the universal element only being used once in any definition. In the 2-valued case, Sobociński [8] has shown that there exists such a functor with four arguments and several authors have developed the 2-valued situation in detail. (See [1], [2], [4], [5], [6].) In the 3-valued case, Rose [7] has given a particular universal element as have the authors in [3].

In the present paper we define a seven input device which will act as a ternary universal decision element. In addition to the variables and the constants 0, 1, 2 we assume that we have both the Post negations of one of the variables available as inputs. The heart of the element is a two-place functor which may be used to generate all the one-place functions using only the variable, its Post negations and the constants, and not requiring any negations of the output. This compares with Rose's approach [7] except that, for his one-place generator, he uses three given one-place functions which may act on both the inputs and output. It is a considerable improvement over the device in [3] which used ten inputs. The technique for generating the two-place functions is to generate each of the rows of the defining truth table separately and combine them using a modified disjunction. This means that it is straightforward to use the element, not requiring complex table look-up to decide the inputs to generate a particular functor.

0, 1, 2 will be used to denote the three values of the ternary system and they will also be used for the three logical constants of the system. Variables will be denoted by x, y, z, etc. We shall identify the one-place functions by their value sequences, that is $\langle abc\rangle$ is the value sequence for the one-place function Kx which is such that $Kx = a$, b, or c according as $x = 0$, 1, or 2 respectively. This will be written $Kx = \langle abc\rangle$. The two Post negations of x will be denoted by Px and Qx, that is $Px = \langle 120\rangle$ and $Qx = \langle 201\rangle$. Note that $PQx = QPx = x$, $PPx = Qx$ and $QQx = Px$.

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