Notre Dame Journal of Formal Logic
Volume XVII, Number 4, October 1976
NDJFAM

THE AXIOMS FOR LATTICOIDS AND THEIR ASSOCIATIVE EXTENSIONS

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By definition, *cf.*, e.g., [1], p. 23, a latticoid is an algebraic system satisfying the following formulas:[1]

$A1$ $[ab]:a, b \in A .\supset. a \cap b = b \cap a$
$A2$ $[ab]:a, b \in A .\supset. a \cup b = b \cup a$
$A3$ $[ab]:a, b \in A .\supset. a = a \cap (a \cup b)$
$A4$ $[ab]:a, b \in A .\supset. a = a \cup (a \cap b)$

The addition of each (but, obviously, not both) of the following two formulas:

$N1$ $[abc]:a, b, c \in A .\supset. a \cap (b \cap c) = (a \cap b) \cap c$
$N2$ $[abc]:a, b, c \in A .\supset. a \cup (b \cup c) = (a \cup b) \cup c$

as a new axiom to $\{A1; A2; A3; A4\}$ generates two different systems which can be called latticoid with meet-associative law and latticoid with join-associative law, respectively.

In this note it will be shown that, although these three systems are rather weak, their respective axiom-systems can be shortened considerably. Namely, I shall prove that:

Any algebraic system

$$\mathfrak{A} = \langle A, \cup, \cap \rangle$$

where $\cup$ *and* $\cap$ *are two binary operations defined on the carrier set* A, *is either a latticoid or a latticoid with meet-associative law or a latticoid with join-associative law, if it satisfies respectively one of the groups of postulates* (A), (B), *and* (C) *which are given below:*

(A) *For latticoids:*

$B1$ $[abcdf]:a, b, c, d, f \in A .\supset. c \cap ((a \cup b) \cap d) = ((b \cup a) \cap ((f \cap d) \cup d)) \cap c$
$B2$ $[ab]:a, b \in A .\supset. a = (a \cup b) \cap a$

1. Throughout this paper A indicates an arbitrary but fixed carrier set. The so-called closure axioms are assumed tacitly.

Received January 7, 1971