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A PROOF OF SOBOCIŃSKI'S CONJECTURE CONCERNING A CERTAIN SET OF LATTICE-THEORETICAL FORMULAS

THOMAS A. SUDKAMP

In [1], J. Ričan has proven that any algebraic system

$$\mathfrak{A} = \langle A, \cup, \cap \rangle$$

where $\cup$ and $\cap$ are two binary operations defined on the carrier set A, which satisfies the following two postulates

A1 $[abc]:a, b, c \in A\ .\supset.\ (a \cap b) \cup (a \cap c) = ((c \cap a) \cup b) \cap a$

and

A2 $[abc]:a, b, c \in A\ .\supset.\ a = (c \cup (b \cup a)) \cap a$

is a modular lattice. In [2] (*cf.* pp. 311-312, Remark I; pp. 313-314, section **2.2**; and p. 315, section **5**, Remark II), B. Sobociński has proven that if in Ričan's postulate-system we substitute *A2* by

B1 $[ab]:a, b \in A\ .\supset.\ a = (b \cup a) \cap a$

then the resulting system $\{A1; B1\}$ satisfies the conditions of a modular lattice with the probable exception, he conjectures, that the associative laws for $\cup$ and $\cap$ fail to hold.

In this note I shall prove this conjecture using the following algebraic table

$\mathfrak{M}1$

$\cup$	0	α	β	γ	δ	1
0	0	α	β	γ	δ	1
α	α	α	β	1	1	1
β	β	β	β	γ	δ	1
γ	γ	1	γ	γ	1	1
δ	δ	1	δ	1	δ	1
1	1	1	1	1	1	1

$\cap$	0	α	β	γ	δ	1
0	0	0	0	0	0	0
α	0	α	α	0	0	α
β	0	α	β	β	β	β
γ	0	0	β	γ	β	γ
δ	0	0	β	β	δ	δ
1	0	α	β	γ	δ	1

which verifies the axioms *A1* and *B1*, but falsifies *A2*,

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