Notre Dame Journal of Formal Logic
Volume XVII, Number 3, July 1976
NDJFAM

A DEDUCTION SYSTEM FOR THE FULL FIRST-ORDER PREDICATE LOGIC

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In [3] H. Hermes and H. Scholz presented an axiomatization which generates exactly the valid formulas of the restricted first-order predicate logic. One of the features of that axiomatization is its symmetry in the underlying deduction rules. In this paper we shall describe an extension and generalization of the axiomatization given by Hermes and Scholz. The axiomatization in [3] is limited to the derivation of valid formulas of the pure first-order predicate logic. Our deduction system $\mathcal{S}$ is formulated for the full first-order predicate logic with identity, including individual constants and functional variables; furthermore, our system $\mathcal{S}$ provides for the deduction of formulas from sets of formulas. The strong completeness and soundness of our system $\mathcal{S}$ guarantees that in $\mathcal{S}$ those and only those deducibility relations are generated which are consequence relations. A deducibility concept, formulated in [1] and in some aspects resembling our formulation, requires a fairly complicated modification in order to obtain soundness (or even the validity of the Deduction Theorem). A preliminary version of our system $\mathcal{S}$ was first presented at the IV'th International Congress for Logic, Methodology, and Philosophy of Science; see [4].

1 The *vocabulary* for the full first-order predicate logic contains (i) a denumerable set of individual variables, (ii) a countable (i.e., finite or denumerable) set of individual constants, (iii) for each integer $n > 0$ a countable set of n-ary functional variables, (iv) for each integer $n \geqslant 0$ a countable set of n-ary predicate variables, (v) the identity symbol $\equiv$, (vi) the propositional connectives $\sim$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, (vii) the quantifiers $\forall$ and $\exists$, and (viii) the parentheses (,). The set of *terms* is the smallest set which contains the individual variables and constants and which with any n-ary functional variable f and any n terms $t_1, \ldots, t_n$ also contains $ft_1 \ldots t_n$. Atomic formulas are the 0-ary predicate variables, all expressions of the form $pt_1 \ldots t_n$ where p is any n-ary predicate variable and $t_1, \ldots, t_n$ are any n terms, and all expressions of the form $t_1 \equiv t_2$ where t_1 and t_2 are any terms.

Received June 3, 1973