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MODALITY AS A META-CONCEPT

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1 In this paper, we will construct and consider some basic properties of a certain modal logic, or, more correctly, family of modal logics, generated by a construction which is a standard linguistic tool, but has never been applied to modal logics before. All modal logics till now (with the exception of perhaps Lemmon's S0.5) have tried to serve two purposes at once:

i) To establish which truths are necessary, e.g., $\vdash_{\mathsf{T}} p \vee \sim p$ and hence $\vdash_{\mathsf{T}} L(p \vee \sim p)$ by the rule of necessitation.
ii) To establish the relationships between such necessary truths, e.g., $\vdash_{\mathsf{T}} L \sim p \supset \sim Lp$.

In other words, the axioms of the logic have had to be sufficient to establish modal truths such as $\vdash_{\mathsf{T}} L(p \supset q) .\supset. (Lp \supset Lq)$ and non-modal truths, such as the theses of the propositional calculus. We will show in this paper that these functions are better separated. Further, it is surprising that it has never been fully realized that modality is a meta-concept; for 'necessary' as applied to sentences/propositions is in fact a convenient corruption of 'necessarily true' (and similarly for 'contingent'). The sentence/proposition is not itself necessary: it could have been any other. The necessity for being what it is belongs to its truth-value. Now, we know very well that for an adequate formalization of the concept of truth, we have need of a meta-language, we need then, *a fortiori*, a meta-language for the formalization of the concept of necessary truth.

A glimpse of the danger of telescoping the object and meta-language is the following. Consider the sentence:

(A) 'This sentence is not necessarily true'. (Or its Quinean version 'gives a statement that is not necessarily true when appended to its own quotation'.)

Suppose this is false, then it is necessarily true, and hence true. Consequently, (A) cannot be false. Hence it must be true. Thus it is not necessarily true and must be contingently true. It must, therefore, be possible for it to be false, but as we have seen this is not the case.

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