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EFFECTIVE DENSITY TYPES

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1 *Introduction* Recursive density types were introduced in [6] and studied in [4] and [5]. It was found that algebraic operations can be defined on the set of types so that the set becomes an interesting algebraic system roughly analogous to the algebra of recursive equivalence types. Further results of a more model-theoretic nature were announced by P. Aczel in [1] and [2]. (The details of [2] occur in a manuscript in our possession which is unpublished as far as we know.*) The main result in [2] is that certain ideals discussed in [4] and [5] satisfy the same universal sentences as the isols. (This is not quite precise as the language is richer, e.g., $\leqslant$ is a primitive relation, and a larger class of functions than the recursively combinatorial functions are allowed.) All this suggests that a further study of the model-theoretic properties of the recursive density types would be of interest. So far, it seems that the algebra of density types enjoy some saturation properties, and on the basis of work by Nerode and Barback the types look more like regressive isols than arbitrary isols. We hope to study all this in future papers.

In this paper, which is still at a pre-model-theoretic level we plan to study a subsystem of the algebra of density types which was motivated by [3]. This will add a new system to the systems Δ_s, Δ_u, and Δ_t studied in [4] and [5]. In [3] Arslanov introduced the concept of effectively hyperimmune set and studied the properties of such sets. (Actually he was interested primarily though not exclusively in effectively hypersimple sets.) In this paper we show that effectiveness is a property of the density type only so that one can introduce the concept of effective density type. Furthermore we show that the set of effective density types forms an ideal properly included in Δ_s though not containing Δ_u. In particular, effectiveness is sufficient to guarantee that such types satisfy the cancellation law as stated in Theorem 5 in [4]. Thus the concept of effectiveness leads to a purely algebraic consequence! This is what generated the interest in studying effective density types.

*Added in proof: P. Aczel, "Recursive density types and Nerode extensions of arithmetic," *Journal of the American Mathematical Society*, Series A, vol. 20 (1975), pp. 146-158.

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