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NOR LOGIC: A SYSTEM OF NATURAL DEDUCTION

LAURENCE S. GAGNON

It has long been known that classical sentential logic can be based on either **NAND** or **NOR** operations. Only recently, however, has a natural deduction system for **NAND** been developed by Price [1]. In a similar vein, the aim of this paper is to present a consistent and complete set of inference rules for the **NOR** operator. The metalinguistic notation used is basically that of Goodstein [2].

The present **NOR** system contains an introduction rule and two elimination rules, each of which has two forms.

$$\text{Xi:}\quad \frac{\ldots A \vdash B,\ \ldots A \vdash B \downarrow B}{A \downarrow A}$$

$$\text{Xe:}\quad \frac{A \downarrow B,\ A}{C} \qquad \frac{A \downarrow B,\ B}{C}$$

$$\text{XXe:}\quad \frac{(A \downarrow B) \downarrow (A \downarrow B),\ A \downarrow A}{B} \qquad \frac{(A \downarrow B) \downarrow (A \downarrow B),\ B \downarrow B}{A}$$

The introduction rule, Xi, is the only one of the set which allows one to discharge an assumption (hypothesis) from a proof. Since the standard matrix for **NOR** validates all of the inference rules, the system is consistent. Moreover, the rules are independent of one another as can be seen by the following reinterpretations of the **NOR** operator.

(1) If $A \downarrow B$ is reinterpreted in terms of the classical matrix for $\neg(B \rightarrow A)$, then Xe and XXe are valid but Xi is not.

(2) If $A \downarrow B$ is reinterpreted in terms of the classical matrix for $\neg(A \,\&\, B)$, then Xi and XXe are valid but Xe is not.

(3) If $A \downarrow B$ is reinterpreted as follows, where 1 is the designated value, then Xi and Xe are valid but XXe is not.

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