CONSISTENCY OF n-ORDER LOGICS

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1 *Introduction** A very natural second-order generalization of first-order predicate logic might be expected to result from an application of the intuition that predicate letters, and perhaps function letters, can be treated in much the same fashion as individual variables. The content of this intuition would seem to be that predicate letters are considered a kind of variable (rather than a kind of constant, as in the usual treatment of first-order logic). Both axioms and rules of inference which affect quantification of individual variables may be extended to license the same operations on predicate letters. Finally, predicate letters are permitted to appear in other than the initial positions of atomic well-formed formulae ("wfs" hereinafter). This last provision may, in some accounts, be accompanied by the introduction of predicate letters of higher type (predicates of predicates) which occur only in the initial position, and over which quantification is not permitted.

Motivating arguments for higher order extensions of predicate logic usually proceed by producing an example of a clearly valid argument which seems to be most naturally rendered schematically by means of the higher order apparatus. "Richard has all of George's good qualities. Candor is a good quality. George is candid. Therefore, Richard is candid," would seem a fairly typical example.

Formal treatments of higher order logics tend to be concerned with disguised pieces of set theory, with comprehension axioms and the other trappings of set theory, or they tend to assume the restrictions of a type theory, or both. Informal treatments, like that in Copi [1], are too vague in their specification of the generalization to permit meaningful discussion of the consistency question. Quine, in [3], has charged that any "natural" (in our sense) generalization must be inconsistent.