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A FIRST-ORDER LOGIC OF KNOWLEDGE AND BELIEF WITH IDENTITY. I

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In establishing the semantic completeness of a first-order system, we customarily show how to generate, from a given hypothetically unprovable formula, a set of formulae which provide, in a rather direct way, a countermodel for the given formula. The "model sets" obtained by the completeness procedure make possible a syntactic treatment of semantics.

By generalizing the notion of model set to that of "model system," Hintikka [1] has been able to provide insight into the logic of knowledge and belief. However, his informal approach tends to obscure the underlying semantical assumptions. In this paper, Hintikka's informal, partly syntactic, partly semantic notion of model system is analyzed into the syntactic and semantic components of a formal first-order Gentzen-type system. With the semantics plainly open to view, it appears that some of the difficulties [2] encountered in Hintikka evaporate or are at least to be located elsewhere. In Part II the system is shown to be semantically complete.

1 *Language of* $\mathcal{J}\langle \mathsf{K}, \mathsf{B}\rangle$ The primitive basis of $\mathcal{J}\langle \mathsf{K}, \mathsf{B}\rangle$ consists of the seven improper symbols

$$\mathsf{N}\;\mathsf{C}\;\mathsf{E}\;\mathsf{K}\;\mathsf{B} \rightarrow ,$$

and the following proper symbols:

(1) the 1-ary functional constant P;
(2) the 2-ary functional constant I;
(3) an infinite set F of free individual variables;
(4) an infinite set B of bound individual variables;
(5) an infinite set of propositional variables; and
(6) for each n, an infinite set of n-ary functional variables.

We shall not specify the contents of these sets. However, as usual, we shall assume that they are pairwise disjoint, that no improper symbol or functional constant of $\mathcal{J}\langle \mathsf{K}, \mathsf{B}\rangle$ belongs to any of them, that each is well-ordered (alphabetically), and that membership in each is effectively

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