Notre Dame Journal of Formal Logic
Volume XVII, Number 1, January 1976
NDJFAM

A NOTE ON THE AXIOM OF CHOICE IN LEŚNIEWSKI'S ONTOLOGY

CHARLES C. DAVIS

This paper generalizes the results of [1] and hence a familiarity with [1] is presupposed. In [1] it was shown that the formulae:

$$\mathbf{AC}^{\varepsilon}\quad [\exists f] :: [Aa] : A\,\varepsilon\, a\, . \supset . f\lceil a\rceil\ \varepsilon\, a$$
$$\mathbf{ACH}\quad [\exists R] \therefore \rightrightarrows\{R\} : [Aa] : A\,\varepsilon\, a\, . \supset . [\exists B] . B\,\varepsilon\, a\, . R\{aB\}$$

are inferentially equivalent in the field of Leśniewski's Ontology, and further that they are equivalent to standard forms of the axiom of choice. Both $\mathbf{AC}^{\varepsilon}$ and **ACH** are stated using the primitive epsilon of ontology, a functor belonging to the semantical category $\mathsf{S/nn}$.

Here the equivalence result will be explicitly extended to cover generalizations of these two formulae, stated using so-called higher epsilons, functors analogous to the primitive epsilon but belonging to categories of the form $\mathsf{S}/\alpha\alpha$, where α is an arbitrary semantical category.

The paper divides naturally into four parts. Section **1** (**2**) introduces the general form of the definition of the generalized epsilon for nominal (propositional) categories and shows that a thesis having the same structural form as the primitive axiom for Ontology is derivable. Section **3** (**4**) presents the demonstration of the equivalence of $\mathbf{AC}^{\varepsilon}_{\alpha}$ and $\mathbf{ACH}_{\alpha}$, where α is a nominal (propositional) category.

If α is an arbitrary nominal (propositional) category and ϕ is a functor belonging to category α, then $\phi\lceil\nu_1 \ldots \nu_n\rceil$ ($\phi\{\nu_1 \ldots \nu_n\}$) will stand for the expression which belongs to the category $\mathsf{n(S)}$ that has as its first word the functor ϕ and that contains the variables $\nu_1, \ldots, \nu_n$, in that order. Representing formulae this way greatly simplifies the treatment given to them, since the exact structure of the parenthemes associated with a formula plays no role in the demonstrations to follow. Of course the "proofs" that incorporate such formulae are not proofs at all; rather they are proof schema which allow for the construction of genuine proofs for formulae of any determinate category.

The applicability of the proof schema presupposes the availability of certain definitions and theses. In particular the previous definition of

Received October 7, 1974