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Illative Combinatory Logic Without Equality as a Primitive Predicate

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Introduction Combinatory logic has the simplest formal framework of any system of logic or mathematics. It has a finite set of primitive constants (or obs) which include K and S, the basic combinators. There is only one very simple formation rule:

If X and Y are obs so is (XY).

These obs include all the operators, expressions, terms, well-formed formulas, etc., of the system.

Pure combinatory logic has one predicate (=). A predicate in this sense is not an ob, it allows us to make statements about obs.

If X, Y, and Z are obs we assume the following axioms:

$$KXY = X$$
$$SXYZ = XZ(YZ)$$

as well as the rules:

(σ) If $X = Y$ then $Y = X$.
(τ) If $X = Y$ then $Y = Z$ then $X = Z$.
(μ) If $X = Y$ then $ZX = ZY$.
(θ) If $X = Y$ then $XZ = YZ$.

Illative (i.e., applied) combinatory logic has additional constant obs (which could also be part of any pure system) and it has, in its usual form, an additional predicate $\vdash$, as well as some axioms and rules involving the new obs = and $\vdash$.

Having listed the primitive concepts of a formal theory many authors would then attempt to provide a semantics, i.e., they would explain these

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