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Generalized S2-Like Systems of Propositional Modal Logic

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In the usual semantics for *S2*, the models are permitted to contain "abnormal" worlds, i.e., possible worlds meeting two conditions: (1) they are terminal (no worlds are accessible from them), and (2) at the abnormal worlds the usual truth conditions for $\Box p$ and $\Diamond p$ are replaced by the conditions that $\Box p$ be false and $\Diamond p$ be true.[1] If a normal world were terminal, then $\Box p$ would be vacuously true at that world and $\Diamond p$ trivially false. Hence we might consider that abnormal worlds are ones at which the usual meanings of the two modal operators are simply interchanged, and that in *S2* it happens that such abnormal worlds are always terminal. This way of looking at *S2* semantics suggests that it might be of some interest to weaken the conditions on a model for modal logic by permitting worlds like *S2*'s abnormal worlds without demanding that they be terminal. Then models for *S2* and its various extensions might be seen as special cases of models of this very general sort.

For each model of this sort, there would be a complementary model, in which there would be abnormal worlds at all those "places" at which the first model had normal worlds, and vice versa, so that in the two models taken as wholes the meanings of the two modal operators would be exactly reversed. The collection of all modal propositions which hold in all such models must therefore be closed under interchange of $\Box$ and $\Diamond$. Indeed it seems likely that this collection of propositions will consist of just those theorems of *S2* which would remain theorems of *S2* under an interchange of modal operators.

In the present paper we shall establish completeness results for a variety of systems of propositional modal logic employing such models. Both the style of presentation and the methods of proof we shall employ owe much to the work of E. J. Lemmon and Dana Scott [5]. In that work, however, Lemmon and Scott do not consider models involving abnormal worlds.[2]

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