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Elementary Formal Systems as a Framework for Relative Recursion Theory

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1 Background on elementary formal systems A nonempty ordered finite set K is called an alphabet. Members of this set are called symbols. By a word in K, we mean a nonempty finite sequence of symbols of K. Given the n symbols $x_1, x_2, \ldots, x_n$ of K (not necessarily distinct), let $x_1x_2 \ldots x_n$ be the word in K whose i^{th} symbol is $x_i (1 \leqslant i \leqslant n)$. The length of a word is the number of symbols (counting repetitions) in that word. If X and Y are words in K, and X is the word $x_1x_2 \ldots x_n$, Y is the word $y_1y_2 \ldots y_m$, then XY is also a word in K and XY is the word $x_1x_2 \ldots x_ny_1y_2 \ldots y_m$. XY is called the concatenation of X and Y.

Definition of an elementary formal system (EFS) By an elementary formal system (E) over an alphabet K, we mean a collection of the following:

1. the alphabet K
2. another alphabet of symbols called variables, which range over words in K
3. another alphabet of symbols called predicates, each of which is assigned a unique positive integer called its degree
4. two more symbols called the implication sign and the punctuation sign
5. a finite sequence $A_1, \ldots, A_n$ of strings which are well-formed formulas, called axioms. (The rules for their formation are listed below.)

The alphabets in 1-4 are to be mutually disjoint. Elements of K usually are denoted by 'a', 'b', etc., variables by 'x_1', 'x_2', etc., or 'x', 'y', etc., and predicates by 'P', 'Q', etc., sometimes with superscripts and subscripts. The implication sign and the punctuation sign are denoted, respectively, by '$\rightarrow$' and ','.

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