Notre Dame Journal of Formal Logic
Volume 23, Number 1, January 1982

On the Number of Nonisomorphic Models in $L_{\infty,\kappa}$ When κ is Weakly Compact

SAHARON SHELAH*

In a previous paper [3] we proved that if $V = L$ then for every regular cardinal λ which is not weakly compact and any model M of cardinality λ, the number of nonisomorphic models of cardinality λ which are $L_{\infty,\lambda}$-equivalent to M is 1 or 2^λ. Here we are going to prove that the above theorem is not true for λ weakly compact.

Main Theorem *Let λ be a weakly compact cardinal. Then there exists a model M, $\|M\| = \lambda$ such that $|K_M^\lambda| = 2$, where $K_M^\lambda = \{N/\cong : N \equiv_{\infty,\lambda} M, \|N\| = \lambda\}$; moreover, we can obtain any number $\leqslant\lambda$ instead of 2.*

Proof: The theorem follows immediately from the next two lemmas.

Notation: We shall always assume that the universe of models of cardinality λ is λ and for $A \subseteq \lambda$ we denote by M_A the submodel of M whose universe is A with the relation symbols R of M of $<|A|$ places such that $R \restriction A \neq \phi$. (Note that e.g., $M \equiv_{\infty,\lambda} N$ does mean that the models have the same language whereas $M \prec_{\omega_1,\omega} N$ does not.)

Lemma 1 *Let M^1, M^2 be models with the following properties:*

(1) $M^1 \not\cong M^2$
(2) $M^1 \equiv_{\infty,\lambda} M^2$
(3) $\|M^1\| = \|M^2\| = \lambda$

*This research was partially supported by the United States-Israel Binational Science Foundation by a grant No. 1110.

Received November 3, 1980