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The n-adic First-Order Undefinability of the Geach Formula

R. E. JENNINGS, P. K. SCHOTCH, and D. K. JOHNSTON*

By adopting natural generalizations of relational frame and relational model we showed in [2] that the deontic law D, $\Box p \rightarrow \Diamond p$, is not universally first-order definable. In effect, we showed there that for $n > 2$, there is no n-adic first-order sentence β such that for every n-ary frame F, $F \models D$ iff $F \models \beta$ in the first-order sense.

The notion of n-ary frame and model employed there may be summarized as follows: $F = \langle U, R \rangle$ is an n-ary relational frame iff U is a nonempty set and R is an n-ary relation on U. Valuations on F are classical for PC formulas. For modal formulas,

$$V(\Box\alpha) = \{x \mid \forall y_1 \ldots y_{n-1}, xRy_1 \ldots y_{n-1} \Rightarrow y_1 \in V(\alpha) \text{ or } \ldots \text{ or } y_n \in V(\alpha)\}.$$

We say that α is valid on F or F is a frame for $\alpha (F \models \alpha)$ iff $V(\alpha) = U$ for every valuation V on F. That this is the correct generalization of frame and model is argued at some length in [3].

Corresponding to the modal notion of a frame is the first-order notion of a model. If F is an n-ary frame and α^* is a sentence in the first-order theory of a single n-adic predicate, then we say that F is a first-order model for $\alpha^* (F \models \alpha^*)$ iff $V(\alpha^*) = 1$ for every assignment of individual variables to objects in F. Taking these notions of frame and first-order model we arrive at the notion of n-adic first-order definability. If there is an n-adic first-order sentence α^* such that for every n-ary frame F, $F \models \alpha^*$ (in the first-order sense) iff $F \models \alpha$, we say that α is *n-adically first-order definable*. If α is n-adically first-order definable for every n, then we say that α is *universally first-order definable*.

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