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Equivalents of a Weak Axiom of Choice

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Some mathematical theorems that have informal proofs involving choices can be proved with at most only a very weak version of the axiom of choice (e.g., The Ball-Game Theorem [12]). In many of these cases the axiom of choice for countable families of finite sets will suffice. This weak axiom of choice is denoted here by AC_ω^f. As an aid in deciding for questionable cases whether any axiom of choice is needed, it may be helpful to note different equivalents of AC_ω^f. Two important combinatorial theorems, Ramsey's theorem and König's infinity lemma, are known to be equivalent to AC_ω^f ([8]; [1], p. 203; [7], p. 298; and [6], pp. 105-106). Here a topological theorem is established as another equivalent of AC_ω^f. Also it is shown that the stronger axiom of choice for arbitrary families of finite sets (to be denoted here by AC^f) is not equivalent to the corresponding stronger topological statement.

For these observations it is assumed that the Zermelo-Fraenkel set theory, ZF, is consistent. Recall that the axiom of choice for finite collections of sets is a theorem of ZF ([7], p. 160) while the slightly stronger AC_ω^f is independent of ZF ([7], p. 167; and [2]). The Tychonoff theorem, which says that the product of a family of compact topological spaces is compact in the product topology, is equivalent to the full axiom of choice, while the Tychonoff theorem limited to Hausdorff spaces is equivalent to the weaker prime ideal theorem ([5]; [10], fn 5; [4], pp. 26-27; and [9]). Let TYC_ω^f denote the following still more limited version of the Tychonoff theorem: "For any countable family of finite sets each regarded as a topological space with the discrete topology, the product of the family is compact in the product topology". Let TYC^f be the statement obtained from TYC_ω^f by omitting the condition that the family is countable.

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