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Completeness Theorems for Two Propositional Logics in Which Identity Diverges from Mutual Entailment

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*1 **Introduction*** In [1] Anderson and Belnap devise a model theory for entailment, on which propositional coentailment equals propositional identity. This feature can be reasonably questioned. Here we devise two extensions of Anderson and Belnap's model theory. Both systems, S and T, preserve Anderson and Belnap's results for entailment, but distinguish coentailment from identity.

The system S is the strongest, satisfying the following plausible principle for propositional substitution: If compound sentences express the same proposition and differ only with respect to the interchange of component sentences A and B, then A and B also express the same proposition. We present a model theory for S and prove soundness and completeness. T results from S by adding just an associativity axiom. The principle for propositional substitution cited above does not hold in T. The model theory for S is extended to T, with soundness and completeness again established.

*2 **Preliminaries*** We consider the formal language E_{fde} formulated in [1], Chapter III. The set of purely truth functional formulas (ptfs) consists of propositional variables plus $\sim A$, $(A \ \& \ B)$, $(A \vee B)$ where A and B are ptfs. The set of formulas consists of ptfs plus $(A \rightarrow B)$ where A and B are ptfs.

In the following, L ranges over intensional lattices ([1], p. 193); $\wedge$, $\vee$, and $-$ denote respectively the meet, join, and complementation operations on L; $\leqslant$ denotes the partial ordering relation on L; $A, B, C \ldots$ range over ptfs.

Q is a model for E_{fde} iff $Q = \langle L, s \rangle$, where s assigns elements of L to each of the variables. If Q is a model for E_{fde} then Val_Q assigns elements of L to ptfs as follows:

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