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Elementary Extensions of Recursively Saturated Models of Arithmetic

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A countable recursively saturated model of arithmetic has, up to isomorphism, a unique countable recursively saturated elementary end extension. As one might guess from the countability restriction, this isomorphism is not canonical. Indeed, if $\mathfrak{M}$ is a recursively saturated model of arithmetic and $\mathfrak{N}$ is an isomorphic copy thereof into which we want to embed $\mathfrak{M}$ as an elementary initial segment, then there are continuum many positions in $\mathfrak{N}$ in which to place $\mathfrak{M}$. In this paper we note that, in fact, $\mathfrak{M}$ can be so embedded in $\mathfrak{N}$ in continuum many decidedly distinct ways.

We are concerned with structures of the form,

$$(\mathfrak{N};\mathfrak{M}) = (|\mathfrak{N}|;|\mathfrak{M}|;+,\cdot,',0,\ldots),$$

where $\mathfrak{M} \prec_e \mathfrak{N}$ are countable recursively saturated models of PA. Our main theorem asserts great variety: for fixed $\mathfrak{M}$ (or, equivalently, for fixed $\mathfrak{N}$) there are continuum many elementarily inequivalent such structures. If, however, we assume $(\mathfrak{N};\mathfrak{M})$ to be recursively saturated, the situation becomes more tractable: there is only a countable infinity of elementarily inequivalent such structures.

Following the introduction, we get down to business in Section 1 where we first discuss cofinal extensions instead of end extensions. The proofs of the corresponding results are simpler and, besides, somewhat cute and deserving of display. End extensions are then discussed in Section 2.

This paper is not self-contained and the reader is advised to have copies of [1] and [6] at hand. Our notation is that of the latter and is reasonably standard. The merely near standard alphabetical exceptions are: Gothic capitals, $\mathfrak{M}$, $\mathfrak{N}$, denote models of arithmetic, which we take to mean *nonstandard* models of arithmetic. Lower case Latin letters are generally integers of various kinds:

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