

## RMLC: Solution to a Problem Left Open by Lemmon

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A system  $S$  is *Halldén-incomplete* if and only if there are wffs  $A$  and  $B$  with no variables in common such that  $\vdash_S A \vee B$  but neither  $\vdash_S A$  nor  $\vdash_S B$ , and *strongly Halldén-incomplete* if, in addition,  $A$  and  $B$  have but one variable apiece.\* Evidently, all strongly Halldén-incomplete systems are Halldén-incomplete; Lemmon [5] poses the converse as an open problem.

Consider the system *RMLC*, with detachment and adjunction as rules and, using standard conventions concerning relative binding strengths of connectives and omission of parentheses, the following axiom schemes:

<b>R0</b>	$A \rightarrow (A \rightarrow A)$
<b>R1</b>	$A \rightarrow A$
<b>R2</b>	$(A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C))$
<b>R3</b>	$A \rightarrow ((A \rightarrow B) \rightarrow B)$
<b>R4</b>	$(A \rightarrow (A \rightarrow B)) \rightarrow (A \rightarrow B)$
<b>R5</b>	$A \ \& \ B \rightarrow A$
<b>R6</b>	$A \ \& \ B \rightarrow B$
<b>R7</b>	$(A \rightarrow B) \ \& \ (A \rightarrow C) \rightarrow (A \rightarrow (B \ \& \ C))$
<b>R8</b>	$A \rightarrow A \vee B$
<b>R9</b>	$B \rightarrow A \vee B$
<b>R10</b>	$(A \rightarrow C) \ \& \ (B \rightarrow C) \rightarrow ((A \vee B) \rightarrow C)$
<b>DUMMETT</b>	$(A \rightarrow B) \vee (B \rightarrow A)$
<b>R11</b>	$A \ \& \ (B \vee C) \rightarrow (\underline{A} \ \& \ B) \vee C$
<b>R12</b>	$(A \rightarrow \overline{B}) \rightarrow (B \rightarrow \overline{A})$
<b>PRE TRANS</b>	$(A \rightarrow (\overline{B} \rightarrow A)) \rightarrow (A \rightarrow (\overline{A} \rightarrow B))$
<b>RMLC</b>	$(\overline{A} \rightarrow A) \vee (B \rightarrow (C \rightarrow B)).$

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