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Automorphisms of ω-Cubes

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1 Preliminaries The word *set* is used for a collection of numbers, *class* for a collection of sets. We write ε for the set of all numbers, o for the empty set of numbers, card Γ for the cardinality of the collection Γ, and $\mathcal{P}_{fin}(\alpha)$ for the class of all finite subsets of α. If f is a function of n variables, i.e., a mapping from a subcollection of ε^n into ε, we denote its domain and range by δf and ρf respectively. A collection of functions is called a *family*. The image under f of the number n is denoted by f_n or $f(n)$, sometimes by both in the same context. We write $\alpha \sim \beta$ for α equivalent to β, $\alpha \simeq \beta$ for α recursively equivalent to β, and $\alpha \oplus \beta$ for the symmetric difference of α and β. The collection of all recursive equivalence types (RETs) is denoted by Ω, that of all isols by Λ. Moreover, $\Omega_0 = \Omega - (0)$, $\Lambda_0 = \Lambda - (0)$, $\varepsilon_0 = \varepsilon - (0)$. The reader is referred to [4] and [8] for the basic properties of RETs and isols. Let $\langle \rho_n \rangle$ be the canonical enumeration of the class $\mathcal{P}_{fin}(\varepsilon)$, i.e., let $\rho_0 = o$ and

$$\rho_{n+1} = \begin{cases} (a_1, \ldots, a_k), \text{ where} \\ n + 1 = 2^{a(1)} + \ldots + 2^{a(k)}, \\ a_1, \ldots, a_k \text{ distinct.} \end{cases}$$

Put $r_n = \text{card } \rho_n$, then r_n is a recursive function. If σ is a finite set, can σ denotes the *canonical index* of σ, i.e., the unique number i such that $\sigma = \rho_i$. For $\alpha \subset \varepsilon$, $i \in \varepsilon$,

$$[\alpha; i] = \{x \mid \rho_x \subset \alpha \ \& \ r_x = i\}, \ 2^\alpha = \{x \mid \rho_x \subset \alpha\} \text{ so that}$$
$$\alpha \simeq \beta \Rightarrow (\forall i)[[\alpha; i] \simeq [\beta; i]], \ \alpha \simeq \beta \Rightarrow 2^\alpha \simeq 2^\beta.$$

If f is a function of one variable, $\delta f^* = 2^{\delta f}$, $f^*(0) = 0$ and

$$f^*(2^{a(1)} + \ldots + 2^{a(k)}) = 2^{fa(1)} + \ldots + 2^{fa(k)},$$

for distinct elements $a_1, \ldots, a_k$ of δf. Equivalently,

$$\delta f^* = 2^{\delta f}, \ \rho_{f^*(x)} = f(\rho_x).$$

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