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On Extensions of $L_{\omega\omega}(Q_1)$

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Introduction $L_{\omega\omega}(Q_1)$ is the logic that results by adding to first-order logic the quantifier "there are uncountably many", studied by Mostowski, Fuhrken, Vaught, and Keisler. It is countably compact and satisfies a downward Löwenheim-Skolem theorem down to $\aleph_1$ (see [3] and [5]). However, it does not satisfy the interpolation theorem (see [4]). An important unsolved problem about this logic is the existence of countably compact extensions of $L_{\omega\omega}(Q_1)$ satisfying interpolation (see [8] and the discussion in [2], p. 221). There are many interesting countably compact extensions of $L_{\omega\omega}(Q_1)$, some of them satisfying the Löwenheim-Skolem theorem down to $\aleph_1$, for example its closure under Δ-interpolation (cf. [1], [8]) or stationary logic, $L_{\omega\omega}(aa)$, a fragment of second-order logic introduced by Barwise, Kaufmann, and Makkai [2]. But none of the known examples satisfies interpolation.

In this note* we show that the monadic fragment of $L_{\omega\omega}(Q_1)$ satisfies the interpolation theorem and is, in fact, a maximal monadic logic satisfying countable compactness and a form of the downward Löwenheim-Skolem theorem down to $\aleph_1$. This is similar to Lindstrom's theorem for $L_{\omega\omega}$, and it follows from the topological properties of the space of models. We introduce monadic *filter*[1] quantifiers and show that they are essentially the cardinal quantifiers (Section 2). A *back-and-forth* characterization of elementary equivalence is given for those logics obtained by adjoining filter quantifiers to the propositional connectives (Section 3). This is used in Section 4 to show that if two sentences of $L_{\omega\omega}(Q_1)$ have an interpolant in the infinitary logic $L_{\infty\omega}(Q_1)$ allowing conjunctions of arbitrary sets of formulas, then they have an interpolant[2] in $L_{\omega\omega}(Q_1)$. Actually, a stronger result is proved: if L^* and $L^\#$ are

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