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On the Number of Nonisomorphic Models of Cardinality λ $L_{\infty\lambda}$-Equivalent to a Fixed Model

SAHARON SHELAH

A well-known result of Scott [6] is that if $\mathfrak{M}$ and $\mathfrak{N}$ are countable and $\mathfrak{M} \equiv_{\infty\omega} \mathfrak{N}$, then $\mathfrak{M} \cong \mathfrak{N}$. Later, Chang [2] extended this to show that if $cf(\lambda) = \aleph_0$, $\mathfrak{M}$ and $\mathfrak{N}$ have cardinality λ and $\mathfrak{M} \equiv_{\infty\lambda} \mathfrak{N}$, then $\mathfrak{M} \cong \mathfrak{N}$. More recently, Palyutin [5] has shown that if $V = L$, $\mathfrak{M}$ has cardinality $\aleph_1$, and $K = \{\mathfrak{N}: \mathfrak{N} \equiv_{\infty\omega_1} \mathfrak{M}$ and $\mathfrak{N} = \aleph_1\}$, then, up to isomorphism, K contains either one member or $2^{\aleph_1}$ members. It has long been known that the first case was not exclusive (cf. [4]).

For $\lambda = \aleph_1$ Palyutin needed the fact that $V = L$ implies $\diamondsuit_S$ for every stationary $S \subseteq \omega_1$. In the Theorem below, we extend Palyutin's result to most other uncountable regular cardinals. Our proof, however, requires a stronger combinatorial principle of Beller and Litman [1] which does not hold in the case of λ weakly compact, and so the restriction in the Theorem.

By Shelah [6] the *GCH* is not enough to guarantee the conclusion even for $\lambda = \aleph_1$, because the "theorem" would imply the following. For λ regular and G a λ-free group of cardinality λ, up to isomorphism $Ext(G, Z)$ has either $\underline{\underline{1}}$ or 2^λ members. However, by [6], "$ZFC + GCH + Ext(G, Z) = Q$ for some G, $\overline{\overline{G}} = \aleph_1$" is consistent.

We now proceed to the theorem and its proof. The result was announced in [8].

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