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An Extension of the Basic Functionality Theory for the λ-Calculus

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1 Introduction The first works about the assignment of types to terms of the λ-calculus (or combinatory logic) arose in the context of logical theories of types.* Church [2] presented a first-order system with types based on λ-conversion, and since then many systems of this kind have been proposed; a review of them is given in the introduction of [15]. The problem of adjoining new objects and deduction rules to combinatory logic is faced in a more general way in [6] and [7], since Curry is interested in studying the properties of illative systems to analyze their suitability as a framework for the study of the foundations of logic. This leads him to introduce many different systems in which the new objects and rules are interesting for different aspects of combinatory logic. As Curry points out ([6], p. 256), the introduction of an illative system is always associated with some interpretation of terms.

The first and simplest of the illative systems proposed in [6] is the theory of basic functionality which is suggested by the very natural interpretation of terms as functions from terms to terms. In this system terms are associated (through a set of formal axioms and deduction rules) with functional characters or types. Types are built from a set of basic objects (which are left uninterpreted) by a composition operator F, which builds a new type $F\sigma\tau$ from two types or basic objects σ and τ. If a term has type $F\sigma\tau$ then we can interpret it as a function from terms of type σ to terms of type τ.

Functionality theory is interesting in the foundational program of Curry essentially since the constant F can very naturally be interpreted as implication. In this case a typed term can be seen as the representation of a proof in a natural deduction system.

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