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A Three-Valued Free Logic for Presuppositional Languages

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Peter Strawson revived the topic of presupposition in 1950.* In subsequent writings, he proposed a notion of semantic presupposition along the following lines: a statement *A* presupposes a statement *B* if and only if *A* is neither true nor false unless *B* is true. Now, it is a simple matter to show that this (semantic) relation is distinct from the more traditional notion of semantic entailment (sometimes called "logical implication"). It is another matter to show that presupposition exists in English, though I think this is also easily done. This, however, is not to say that Strawson was right on all—or even some—of the cases he considered, for what any particular statement presupposes is a further matter still; this last matter is to be decided by what I call one's "theory of presuppositions". It is not my aim to offer or to defend any particular theory of presuppositions here. Instead, I am concerned with the formalization of a logic in which the relation of presupposition is a nontrivial relation.

With these things in mind, the problem this paper is addressed to is this: Can a logical calculus be developed which will allow some statements to be neither true nor false if they have a presupposition which fails to be true? I present in this paper an axiomatic, first-order logic; a semantics is provided, and the axioms are proved to be sound and complete with respect to the semantics. The logic allows a statement to be true, false, or neither-true-nor-false. In every case, if all the statements being considered have a truth-value (true or false), the logic gives the same results that classical logic does. More specifically, what is wanted here is a logic that does three things: it must allow

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