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Inexact Geometry

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1 Introduction In Tarski [13] elementary geometry is construed as a first-order theory, in the classical predicate calculus, of two predicates: the ternary predicate of betweenness and the quaternary predicate of equidistance.* A simplified version for the one-dimensional case, using only betweenness (and, of course, equality, which can be considered part of the logic), is given in Roberts [11]. A finite structure (X, b) is a model of this theory iff there exists a real-valued function f on X such that for all $x, y, z \in X$

$$b(x, y, z) \text{ iff } |f(x) - f(z)| = |f(x) - f(y)| + |f(y) - f(z)|.$$

With only one very minor modification, but with equality replaced by 'indifference', Roberts shows that his axiom system becomes a theory of what he calls 'tolerance geometry'. A finite structure (X, i, b) is a model of this theory, and of the theory of indifference as described in, e.g., Roberts [10], iff given any positive real number ε there is a real-valued function f on X such that for all $x, y, z \in X$

$$i(x, y) \quad \text{iff} \quad |f(x) - f(y)| < \varepsilon$$

$$b(x, y, z) \quad \text{iff} \quad |f(x) - f(y)| + |f(y) - f(z)| - |f(x) - f(z)| < \varepsilon.$$

Tolerance geometry is meant to tolerate errors (of measurement, or of perception) smaller than a fixed but arbitrary ε. It is thought to be particularly suitable for the geometry of visual perception. In this context ε could represent

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