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Solution to a Problem of Chang and Lee

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In this note we show that input resolution with paramodulation (IP) is strictly weaker than unit resolution with paramodulation (UP).

First we introduce some notation. A is always an atomic sentence and p, q are always statement letters. $\vdash_X$, for X = IP, UP, I (input resolution), or U (unit resolution), means derivability by means of the rules of X.

We work in a fixed first-order language and consider only ground clauses. E is the set of all clauses of the form $\{\neg t_0 = t_1, \neg A(t_i), A(t_{1-i})\}$ together with all those of the form $\{t = t\}$.

A set L of literals is consistent if $\neg\exists l \in L\ \bar{l} \in L$.

If L is a consistent set of literals and C is a clause we say $L \approx C$ if $L \cap C \neq \emptyset$ or $\exists l_1 \in C\ \exists l_2 \in C\ l_1 \neq l_2 \wedge \bar{l}_1 \notin L \wedge \bar{l}_2 \notin L$.

If C_1 and C_2 are clauses define $[C_2/p]C_1 = C_1$ if $p \notin C_1$, $[C_2/p]C_1 = (C_1 - \{p\}) \cup C_2$ if $p \in C_1$. If S is a set of clauses define $[C_2/p]S = \{[C_2/p]C_1: C_1 \in S\}$.

Substitution lemma *Suppose there is a UP derivation of C_1 from S with no clause containing $\neg p$ and with $\{p\}$ at most as its last clause, then for each C_2 there is a $C_3 \subset [C_2/p]C_1$ such that $[C_2/p]S \vdash_{\overline{UP}} C_3$.*

The proof of the substitution lemma is routine.

Soundness lemma *If L is a consistent set of literals and S a set of clauses then $L \approx S \cup E \Rightarrow S \cup E \nvdash_I \emptyset$.*

Proof: Prove by induction on the length of an input derivation of C from $S \cup E$ that $\exists l \in C\ (l \in L \vee \bar{l} \notin L)$.

Completeness lemma *If S is a set of clauses then there is a consistent set of literals L such that $S \cup E \nvdash_I \emptyset \Rightarrow L \approx S \cup E$.*

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