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A Completeness-Proof Method for Extensions of the Implicational Fragment of the Propositional Calculus

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The traditional proof that the classical propositional calculus (*PC*) is strongly complete (i.e., if $\alpha \models A$, then $\alpha \vdash A$) is based on the notion of a maximal consistent set of formulas, and hence on certain properties of strong (i.e., *PC*-)negation. In this paper* I present a completeness-proof method which does not refer to maximal consistent sets, but only to sets which are: (i) nontrivial (not all formulas are members), (ii) deductively closed (all syntactical consequences are members), and (iii) implication saturated (for all B, $A \supset B$ is a member if A is not a member). If this proof method is applied to logics that contain strong negation, the sets turn out to be consistent with respect to strong negation. I shall first apply the proof method to a specific extension of the implicational fragment of *PC*, and next show that it also applies to the implicational fragment itself and to a large number of logics that are extensions of the implicational fragment. If such a logic is characterized by a semantics, the articulation of an axiomatic system is straightforward (in view of the proof method) and *vice versa.*

The completeness-proof method is especially fit for paraconsistent logics that are based on material implication (see [1]-[6]).[1] Paraconsistent logics are logics according to which at least some inconsistent theories are nontrivial (some sentences of the language are not derivable from the axioms of the

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