Notre Dame Journal of Formal Logic
Volume 21, Number 3, July 1980

Word Problems for Bidirectional, Single-Premise Post Systems

CHARLES E. HUGHES and DAVID W. STRAIGHT

Introduction A *bidirectional, single-premise Post system* is a Post canonical form F where, if $R_1 \to R_2$ is a rule, then $R_2 \to R_1$ is also a rule. One class of bidirectional Post systems, the Thue systems first defined in [7], have been extensively studied. Thue systems with unsolvable word problems were shown to exist by Post [5] and, more recently, Overbeek [4] demonstrated that this class of problems represents every recursively enumerable (r.e.) many-one degree of unsolvability. In this paper we extend Overbeek's result to include bidirectional extensions of Post normal systems, tag systems, and the one-letter systems introduced by Hosken [1].

Post Systems Let Σ be a finite set of symbols and let $Q_1, Q_2, \ldots, Q_n$ be new symbols called *operational variables*. A word over $\Sigma \cup \{Q_1, Q_2, \ldots, Q_n\}$, containing at least one operational variable, is called a *word form*. An *identification* of the operational variables $Q_1, Q_2, \ldots, Q_n$ is a set of pairs $\{(Q_i, W_i) | 1 \leqslant i \leqslant n\}$ where each W_i is a word over Σ. Let $Y \equiv y_1 Q_{i_1} y_2 Q_{i_2} \ldots y_m Q_{i_m} y_{m+1}$ be a word form where $y_1, y_2, \ldots, y_{m+1}$ are words over Σ and $Q_{i_1}, Q_{i_2}, \ldots, Q_{i_m}$ are operational variables. Then Y' is the result of applying the identification $\Phi = \{(Q_i, W_i) | 1 \leqslant i \leqslant n\}$ to Y, denoted Y^Φ, if $Y' \equiv y_1 W_{i_1} y_2 W_{i_2} \ldots y_m W_{i_m} y_{m+1}$.

A *single-premise Post system* $F = (\Sigma, V, P)$ is such that Σ is a finite alphabet, V is a finite set of operational variables, and P is a finite set of rules, each of the form $R_1 \to R_2$, where R_1 and R_2 are word forms. Let W_1 and W_2 be words over Σ. Then W_2 is said to be an *immediate successor* of W_1 in F, denoted $(W_1, W_2)_F$, if there exists some rule of P, $R_1 \to R_2$, and some identification Φ of V such that $R_1^\Phi \equiv W_1$ and $R_2^\Phi \equiv W_2$. W_2 is said to be *derivable* from W_1 in F, denoted $[W_1, W_2]_F$ (or $[W_1, W_2]$, whenever F is understood from context), if there exists a sequence $Y_1, Y_2, \ldots, Y_k$, where $k \geqslant 1$, of words over Σ such

Received November 6, 1978; revised April 23, 1979