A THEORY OF CLASSES AND INDIVIDUALS BASED ON A 3-VALUED SIGNIFICANCE LOGIC

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1 Introduction

1.1 *The need for the theory*   A problem that arises when one tries to introduce individuals into a theory of sets or classes is that of distinguishing the null class from an individual, since both have no members.* The class theory will contain an axiom of extensionality which will identify two classes or individuals if they have exactly the same members. The null class and an arbitrary individual will have no members and hence, by the axiom, be identical.

The difficulty is discussed by Quine [9], pp. 29-32. One way out is to use separate variables for individuals and for classes or to introduce the primitive predicate 'is an individual' into the system. Quine dismisses these as "unwelcome sacrifices of elegance" and says that happily these can be avoided. Quine instead suggests regarding $x \in y$, where y is an individual, as $x = y$. This avoids the problem with the axiom of extensionality because if y and z are individuals $(\mathbf{A}x)(x \in y \equiv x \in z)$ is equivalent to $(\mathbf{A}x)(x = y \equiv x = z)$, i.e., $y = z$. Quine also shows that this implies that an individual is equal to its unit class and says that this does not affect the development of class theory as required for mathematics. But if one takes a material object and forms its unit class, then, according to Quine, this material object would be equal to its unit class, an abstract entity, and this is unsatisfactory.

By taking $x \in y$ as nonsignificant when y is an individual and using a 3-valued significance logic,[1] one can avoid all the problems that have arisen in connection with distinguishing the null class from individuals. The predicate 'is an individual' can be defined in terms of the logic, i.e., $I(x) =_{df} \sim (\mathbf{S}y)S(y \in x)$, i.e., $y \in x$ is nonsignificant for all y, where the variables x and y range over classes and individuals.

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