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GENERAL COMPUTABILITY

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1 *Introduction* A theory of computability consists of a domain A of cardinality greater than one and a definition of computability for multivariable partial functions from A into A. The class $\mathcal{F}$ of all partial functions satisfying that definition is called the class of computable functions. In this paper, we consider some properties of $\mathcal{F}$ which have been of importance in various specific theories. In particular, we consider conjunctions of the following properties:

(1) $\mathcal{F}$ is the closure of a family $\mathcal{G}$ under composition.

(2) $\mathcal{F}$ is closed under piecewise composition.

(3) $\mathcal{F}$ is the closure of a family $\mathcal{G}$ under composition and iteration where the iterate f^∞ of a partial function f is $\lambda x\,[f^n(x)]$, where n denotes $\mu m\,[f^m(x) = f^{m+1}(x)]$

(4) $\mathcal{G}$ is closed under piecewise composition.

(5) $\mathcal{G}$ is finite.

(6) $\mathcal{F}$ contains the test function $\lambda w, x, y, z\,[x \text{ if } w = z; \text{ else } y]$.

(7) $\mathcal{F}$ includes a pairing system, i.e., a set $\{\rho, \sigma, p\}$ of partial functions such that $\rho(p(a, b)) = a$ and $\sigma(p(a, b)) = b$ for all a and b in A.

(8) ρ, σ, and the members of $\mathcal{G}$ are total functions on A.

(9) $\mathcal{F}$ has an indexing; i.e., $\mathcal{F}$ is closed under composition and contains a partial function $*$ such that every member of $\mathcal{F}$ is of the form:

$$\lambda x_1, \ldots, x_n\,[(\ldots(a * x_1) * \ldots * x_n)]$$

for some a in A.

(10) $\mathcal{F}$ has a uniform indexing in the sense that $\mathcal{F}$ has an indexing $*$ such that every member of $\mathcal{F}$ is of the form

$$\lambda x_1, \ldots, x_n[g(x_1, \ldots, x_{n-1}) * x_n]$$

for some total g in $\mathcal{F}$.

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