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A CLASS OF TWO-PLACE THREE-VALUED UNARY GENERATORS

J. C. MUZIO and D. M. MILLER

1 *Introduction* A unary generator is a functor which may be used to define any one-place functor by the substitution of its arguments from a prescribed substitution set, the generator being used once in each definition with no negation of its output. In this paper, we examine the class of two-place three-valued functors which are unary generators relative to the substitution set consisting of the constants, the variable, and the Post negations of the variable. Necessary and sufficient conditions for a functor to belong to this class are established and it is shown the class consists of 216 functors which are equivalent under a set of five transformations.

A universal decision element is one which may be used to define any of the functors of one or two arguments by the substitution of variables or constants in its arguments, the universal element being used exactly once in each definition. Sobociński [11] has shown that for the two-valued case there exists such a functor with four arguments. The two-valued case has been extensively studied (see [1], [4], [7], [8], [9]). Rose [10], Loader [2], [3], and the authors [5] have presented three-valued universal decision elements. In a previous paper, the authors [6] have given a seven-place three-valued functor which acts as a universal decision element. This functor is a composition of a generalized condition disjunction and three identical functors from the class considered in this paper. The same composition applies to each member of the class resulting in a family of functors each behaving as a three-valued universal decision element.

Loader [2] has presented an alternative definition of a universal decision element. The functors considered here are—in his terminology—first-order universal decision elements of the third degree. Loader has shown that for a two-place three-valued functor to be a unary generator the substitution set must contain at least two nontrivial one-place functors. He has further shown, by ennumeration on a computer, that for a substitution set with two nontrivial one-place functors, the Post negations result in 216 functors acting as unary generators as opposed to either zero or twelve for

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