Notre Dame Journal of Formal Logic
Volume XX, Number 4, October 1979
NDJFAM

EQUATIONAL TWO AXIOM BASES FOR BOOLEAN ALGEBRAS AND SOME OTHER LATTICE THEORIES

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In this paper it will be shown that the postulate systems for Boolean algebras and six other lattice theories can be reduced to sets containing only two equational axioms. As far as I know such axiomatizations of the theories under investigation are not mentioned in the literature.

1 For this end we have to prove the following nine Propositions[1]:

PI *An algebraic system* $\langle A, \cup\rangle$ *is a join semilattice, if it satisfies the following two postulates:*

$A1$ $[a]: a \epsilon A .\supset. a = a \cup a$
$A2$ $[abc]: a, b, c \epsilon A .\supset. a \cup (b \cup c) = c \cup (a \cup b)$

PII *An algebraic system* $\langle A, \cap\rangle$ *is a meet semilattice, if it satisfies the following two postulates:*

$B1$ $[a]: a \epsilon A .\supset. a = a \cap a$
$B2$ $[abc]: a, b, c \epsilon A .\supset. a \cap (b \cap c) = c \cap (a \cap b)$

PIII *An algebraic system* $\langle A, \cup, -\rangle$ *is a Boolean algebra, if it satisfies the following two postulates: A1 and*

$C1$ $[abcd]: a, b, c, d \epsilon A .\supset. a \cup (b \cup c) = (-(-c \cup d) \cup -(-c \cup -d)) \cup (a \cup b)$

PIV *An algebraic system* $\langle A, \cup, \cap, O, I\rangle$ *is a lattice with universal bounds, if it satisfies the following two postulates:*

$D1$ $[abc]: a, b, c \epsilon A .\supset. (b \cap (c \cap a)) \cup a = a$
$D2$ $[abcdef]: a, b, c, d, e, f \epsilon A .\supset. ((a \cap (b \cap c)) \cup d) \cup e$
$= ((((c \cap I) \cup O) \cap (a \cap b)) \cup e) \cup ((f \cup d) \cap d)$

PV *An algebraic system* $\langle A, \cup, \cap, {}^{\perp}\rangle$ *is an ortholattice, if it satisfies the following two postulates: D1 and*

1. Throughout this paper, A indicates arbitrary but fixed carrier set. The so-called closure axioms are assumed tacitly.

Received December 8, 1978