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ON FULL CYLINDRIC SET ALGEBRAS

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By a full cylindric set algebra of dimension α, full $\mathbf{CSA}_\alpha$, where α is an ordinal number, we mean a system

$$\mathfrak{A} = \langle A, \cup, \cap, \sim, 0, {}^{\alpha}U, \mathbf{C}_\kappa, \mathbf{D}_{\kappa\lambda}\rangle_{\kappa,\lambda<\alpha}$$

where U is a non-empty set, A is the power set of ${}^{\alpha}U$, 0 is the empty set, $\cup$, $\cap$, and $\sim$ are the set theoretic union, intersection and complement on A, and for all κ, $\lambda < \alpha$, $\mathbf{C}_\kappa$ is a unary operation on A and $\mathbf{D}_{\kappa\lambda}$ is a constant defined as follows:

$$\mathbf{C}_\kappa X = \{y\colon y \in {}^{\alpha}U \text{ and for some } x \in X \text{ we have } x_\lambda = y_\lambda \text{ for all } \lambda \neq \kappa\}$$
$$\text{for every } X \in A,$$

and

$$\mathbf{D}_{\kappa\lambda} = \{y\colon y \in {}^{\alpha}U \text{ and } y_\kappa = y_\lambda\}$$

(*cf.* 1.1.5, [2]). In section **1** we give an axiom system for a subclass of cylindric algebras, which we call strong cylindric algebras, and show that $\mathfrak{A}$ is a strong $\mathbf{CA}_\alpha$, $\alpha < \omega$, if, and only if, $\mathfrak{A}$ is isomorphic to a full $\mathbf{CSA}_\alpha$.

In section **2** we restrict our attention to the theory of strong $\mathbf{CA}_2$ and show that it is definitionally equivalent to the theory of a subclass of relation algebras axiomatized by McKinsey [3].

The notation of [1] is used, and a familiarity with chapter 1 of that book is assumed.

1 *Strong cylindric algebras* We begin by introducing a piece of notation which will prove to be convenient.

Definition 1.1 If $\mathfrak{A}$ is a $\mathbf{CA}_\alpha$, $\alpha < \omega$, and $i < \alpha$, then

$$\mathbf{c}^i x = \mathbf{c}_{(\alpha \sim \{i\})} x.$$

Definition 1.2 By a strong cylindric algebra of dimension α, where α is an ordinal number less than ω, we mean a structure

$$\mathfrak{A} = \langle A, +, \cdot, -, 0, 1, \mathbf{c}_\kappa, \mathbf{d}_{\kappa\lambda}\rangle_{\kappa,\lambda<\alpha}$$

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