

IDENTITY LOGICS

JOHN CORCORAN and STANLEY ZIEWACZ

In this paper* we prove the completeness of three logical systems **IL1**, **IL2** and **IL3**. **IL1** deals solely with identities ($a = b$), and its deductions are the direct deductions constructed with the three traditional rules: (*T*) from $a = b$ and $b = c$ infer $a = c$, (*S*) from $a = b$ infer $b = a$ and (*A*) infer $a = a$ (from anything). **IL2** deals solely with identities and inidentities ($a \neq b$) and its deductions include both the direct and the indirect deductions constructed with the three traditional rules. **IL3** is a hybrid of **IL1** and **IL2**: its deductions are all direct as in **IL1** but it deals with identities and inidentities as in **IL2**. **IL1** and **IL2** have a high degree of naturalness. Although the hybrid system **IL3** was constructed as an artifact useful in the mathematical study of **IL1** and **IL2**, it nevertheless has some intrinsically interesting aspects.

The main motivation for describing and studying such simple systems is pedagogical. In teaching beginning logic one would like to present a system of logic which has the following properties. First, it exemplifies the main ideas of logic: implication, deduction, non-implication, counter-argument (or countermodel), logical truth, self-contradiction, consistency, satisfiability, etc. Second, it exemplifies the usual general metaprinciples of logic: contraposition and transitivity of implication, cut laws, completeness, soundness, etc. Third, it is simple enough to be thoroughly grasped by beginners. Fourth, it is obvious enough so that its rules do not appear to be arbitrary or purely conventional. Fifth, it does not invite confusions which must be unlearned later. Sixth, it involves a minimum of presuppositions which are no longer accepted in mainstream contemporary logic. These are vague conditions which are satisfied to a greater or lesser extent by propositional logic, **PL**, and by Aristotelian logic (or syllogistic), **AL**, and the majority of contemporary beginning logic students are

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