QUANTUM LOGIC WITH IMPLICATION

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The non-classical nature of quantum theory has led to a number of efforts to formalize the logic of statements whose universe of discourse is the subatomic (see, for example, [3], [6], [8], and [10]). It might be hoped that ideally the fruit of such efforts would be a formalism not totally divorced from the classical logic, but incorporating or incorporated in it in such a manner as to unify the logic of the micro- and the macro-worlds. Indeed, this has been the general direction taken by these efforts, including that which we principally examine and expand upon here.* Josef Jauch, [6], pp. 67-89, developed a "propositional calculus of quantum mechanics". He objected to calling a system like his a "*logic* of quantum mechanics"; however, our use of 'logic' here is not, I think, the use to which Jauch objected. A logic, for our present purposes, is a mathematical formalism having a reasonable interpretation as a theory of deduction; its actual application as normative of argumentation in reality is a matter of empirical judgment. This seems close to the way Jauch wished to use "propositional calculus" but more general, allowing for extension to a quantification theory of quantum mechanics, for example.

Although Jauch's presentation of a propositional calculus of quantum mechanics is mathematically interesting and suggestive, there are a number of problems connected with it. It is our purpose to engage some of these problems and to present a rigorously defined formalism intended to come close to doing the job intended by Jauch for his logic. The success or lack of success of this effort as related to actual application of this formalism in the foundations of quantum theory is, I think, a matter for the physicists to decide; the empirical judgment involved in this application is not in the scope of this paper.

If a formalism is to have a reasonable interpretation as a theory of deduction, it seems clear that it must include a relation which, when it holds between two elements of the formalism, may be reasonably interpreted as expressing that one of the elements 'follows from' the other, in

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