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NOTE ON A STRONG LIBERATED MODAL LOGIC AND ITS RELEVANCE TO POSSIBLE WORLD SKEPTICISM

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We consider a modal language with a set of sentence parameters $P = \{P_1, P_2, \ldots\}$ and the symbols L (necessity), $\supset$, and $\neg$ for sentential connectives. We will consider axiomatic systems based on the following axiom schemes:

A1 All tautologies
A2 $L(E \supset E') \supset (LE \supset LE')$.

As rules of proof, we assume the following:

MP If E and $E \supset E'$ are provable, then so is E'.
Nec If E is provable, then so is LE.

Consider the two following axiom schemes:

A3 $LE \supset E$
A4 $E \supset LE$.

If we add both A3 and A4 to our basic system, we obtain the trivial system in which for any expression E, LE is equivalent to E. The classical systems adopt A3 and exclude A4 to obtain **T** and stronger systems (see [1]). Murungi [6] has given a formal proof of the claim that the system obtained by adding A4 to the basic system, sketched above, is consistent. The system with A4 is called **T'**.

While it is true that **T'** is consistent and does not collapse into the trivial system, **T'** comes *very* close to collapsing into the trivial system, as we shall show by providing a semantics and completeness proof for **T'**. The system **T'** turns out to be one of the so-called liberated modal logics. It is a very strong logic containing all of the liberated systems discussed in [2]-[5].

The semantic range is the usual $\{\mathbf{t},\mathbf{f}\}$. We designate by G an indexed set of functions g mapping the set of sentence parameters P into the semantic range. An interpretation is an ordered triple (g, W, R) where: $g \epsilon G$; $W \subseteq G$; R is a binary relation (accessibility) defined on W; if W is not empty, then

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