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# ON THE NECESSITY OF S4

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In [4] I pointed out that Lewis' verbal definitions of necessity and impossibility in [2], pp. 248-249 constitute an essential part of his famous "Independent" proofs. For ease of reference I quote Lewis' words again:

> To say '$p$ is necessary' means '$p$ is implied by its own denial' or 'the denial of $p$ is not self-consistent' . . . . To say '$p$ is impossible' means '$p$ implies its own denial' or '$p$ is not self-consistent'. Necessary truths so defined coincide with the class of tautologies or truths which can be certified by logic alone; and impossible propositions coincide with the class of those which deny some tautology.
>
> Every tautology is expressible as some proposition of the general form $p \vee -p$. . . . The negation of any proposition of the form $p \vee -p$ is a corresponding proposition of the form $p \,.\, -p$.

I extracted the following symbolic definition of impossibility from this passage:

(i) $-\Diamond p =_{df} [p = (r \,\&\, -r)]$, see [4], p. 545.

By substituting $-p$ for $p$ we obtain the corresponding definition for *necessity*:

(ii) $-\Diamond -p =_{df} [-p = (r \,\&\, -r)]$

If in (i) we negate both sides of the definitional equality and apply *Double Negation* (strong version) and *Substitution*, we have

(iii) $\Diamond p =_{df} -[p = (r \,\&\, -r)]$

It is obviously feasible to treat (iii) as primitive and the others as derivative. The sign '=' is used here in the *strict* sense of Lewis in which

$$p = q =_{df} [(p \prec q) \,\&\, (q \prec p)]$$

I wish in this paper to prove that if (iii) is added to Lewis' system S1, then, using a certain strong but very plausible version of the principle of the substitutivity of strict equivalents to be discussed below, we can obtain S4. (See [2], pp. 123-153, where S1 is established and developed). In the impending exercise, I write '$M$' for "it is possible that" and '$L$' for "it is

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