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ALTERNATIVE FORMS OF PROPOSITIONAL CALCULUS FOR A GIVEN DEDUCTION THEOREM

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In a propositional calculus based on combinatory logic it is necessary to have a restriction on the deduction theorem for implication as otherwise Curry's paradox results (see [5]). In [1] and [2] we restricted the deduction theorem for implication as follows:

DTP *If* $\Delta, X \vdash Y$, *then* $\Delta, \mathsf{H}X \vdash X \supset Y$,

where Δ is any sequence of obs and $\mathsf{H}X$ stands for "X is a proposition".

Motivation for this deduction theorem was given in [2] using the following three valued tables (that for implication also appears in Kleene [6]).

X	$\mathsf{H}X$
T	T
F	T
N	N

X	ΓX
T	F
F	T
N	N

	$X \supset Y$	Y: T	F	N
X	T	T	F	N
	F	T	T	T
	N	T	N	N

where N can stand for "neither T nor F" and Γ (negation) can be defined by $\mathbf{CP}(\Xi\mathbf{HI})$,[1]

A question that arises is: to what extent are the entries in the third column and the third row of the table for implication uniquely determined by DTP, modus ponens and the (fairly obvious) rule:

H $X \vdash \mathsf{H}X$?

1. Here $\mathbf{P}$ stands for implication. $\Xi\mathbf{HI}$, which can be interpreted as stating that all propositions are provable, is taken as the "standard false" proposition. Given that $\Xi\mathbf{HI}$ is assigned F the table for Γ follows from that for $\supset$.

After part (iii) on page xx, below we assume for $\Xi\mathbf{HI}$:

$$\Xi\mathbf{HI}, \mathbf{H}X \vdash X.$$

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