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A MATRIX DECISION PROCEDURE FOR THREE MODAL LOGICS

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The decision procedure described below was reached by an attempt to define a 'decidability' operator, 'U', that would clarify the interpretation of nested modalities. '$U\alpha$' was intended to mean:

(the truth value of) α is (non-referentially) decidable.

I.e. We need not know what proposition (statement) α is in order to know whether it is true or false. Note that I use metalogical variables such as 'α' as abbreviations for (not names of) logical formulae, but I use logical formulae such as 'p' as names of propositions; I therefore treat operators such as 'K' as generators of names of (complex) propositions from names of propositions.

The 'necessity' operator, 'L' is defined thus:

(Def L) $L\alpha =_{df} K\alpha U\alpha$.

If 'L' is taken as primitive, 'U' may be defined thus:

(Def U) $U\alpha =_{df} AL\alpha\ LN\alpha$.

The system **U11** **U11** is intended to imitate the propositional calculus (**PC**) in regarding as decidable just what is formally decidable within the system itself, the concept of decidability being imported from the metalogic of **PC** into the logic of **U11**.

For any wff, f, we may generate a matrix (truth table) having c columns and r lines, where f has c letters including n distinct variables and $r = 2^n$. Each line contains a distinct assignment of truth values to the variables (0-false, or 1-true). We may express the internal relations of the propositions named by the variables of f as a vector, **v**, of length r, containing the values 0 and 1, the k'th element of **v** indicating that the internal relations of the propositions do (1) or do not (0) permit the assignment of truth values in the k'th line of the matrix. E.g. for some wff '$\ldots p \ldots q$', if Epq is true then of the lines:

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