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THE APPROACHES TO SET THEORY

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Introduction

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Our aim is to consider and criticise the different approaches to set theory; we often seem to disagree with the accepted views.

We describe Cantor's work in chapter **1**. Then we emphasize its second order nature, indicate how it seems to have been misunderstood and suggest that a lot of later work was motivated by such misunderstandings. Part of chapter **2** gives a justification of **ZF** in Cantorian terms and in the remainder of that chapter we consider related problems and quasi-constructive approaches. In chapter **3** we consider set theories with a universal set; the most well known of these being **NF**. Other sections of that chapter concern approaches via theories of properties.

CHAPTER 1—CANTOR'S WORK

1.1 *Introduction* Although it seems possible to trace the notion of a set back for an indefinite period, it is indisputable that Cantor's work made the greatest step, by far, in the development of the idea. This is one of the reasons why we think it important to consider his work here. The other is that its nature is often misrepresented in textbooks and mythology today. Basically we shall give an account of Cantor's work on the notion of a set and, from his publications, we can discern three stages in the development of his ideas. It is quite possible that Cantor's views remained constant and that we are really only considering different stages of presentation, but we shall always write as if his papers correspond to his ideas. The main references which we shall use are [5], [6], [7] and [17], and we shall usually refer to Cantor's papers just by the year in which they were first published.

As well as describing Cantor's ideas we shall often comment on points at which various problems arise and sometimes we shall investigate them further. Also, we shall try to show how, in the development of set theory,

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