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POINT MONADS AND P-CLOSED SPACES

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1 *Introduction** Let P be any topological property. Recall that a space (X,τ) is *P-closed* if X is a P-space and a closed subset of every P-space in which it is embedded. As is well known [1] for P = completely regular, normal, paracompact, metric, completely normal, locally compact, zero-dimensional, a P-space is P-closed iff it is compact. Robinson [12] was the first to show that a space (X,τ) is compact iff ${}^*X = \bigcup\{\mu(p) \mid p \in X\}$, where $\mu(p) = \bigcap\{{}^*G \mid p \in G \in \tau\}$. In [7], [9], it is shown that a space (X,τ) is Hausdorff-closed (henceforth called H-closed) iff ${}^*X = \bigcup\{\mu_\theta(p) \mid p \in X\}$, where $\mu_\theta(p) = \bigcap\{{}^*(\mathrm{cl}_X G) \mid p \in G \in \tau\}$. A space (X,τ) is almost completely regular [13] if for each regular-closed $A \subset X$ (i.e., $A = \mathrm{cl}_X \mathrm{int}_X A$) and $x \notin A$ there exists a real valued continuous map $f: X \to [0,1]$ such that $f[A] = \{0\}$ and $f(x) = 1$. In [9], it is shown that an almost completely regular Hausdorff space (X,τ) is almost completely regular-closed iff ${}^*X = \bigcup\{\mu_\alpha(p) \mid p \in X\}$, where $\mu\ (p) = \bigcap\{{}^*(\mathrm{int}_X \mathrm{cl}_X G) \mid p \in G \in \tau\}$. The monad $\mu(p)$, α-monad $\mu_\alpha(p)$ and θ-monad $\mu_\theta(p)$, in addition to characterizing various P-closed spaces, are extensively employed to investigate numerous other important topological properties. Of particular interest is the result in [6] which shows that a filter base $\mathfrak{F}$ on X is Whyburn [resp. Dickman] iff $\mathrm{Nuc}\ \mathfrak{F} = \bigcap\{{}^*F \mid F \in \mathfrak{F}\} \subset \mathrm{ns}({}^*X) = \bigcup\{\mu(p) \mid p \in X\}$ [resp. $\mathrm{Nuc}\ \mathfrak{F} \subset \mathrm{ns}_\theta({}^*X) = \bigcup\{\mu_\theta(p) \mid p \in X\}$]. For other recent results using these monads, we direct the reader to references [6], [7], [8], [10], [11]. Elementary applications of the α and θ-monads and simple basic propositions may be found in [7].

The major goal of this paper is to define a new monad, the w-monad, and show that it characterizes the completely Hausdorff-closed spaces in the usual nonstandard manner. A space (X,τ) is *completely Hausdorff* (sometimes called Urysohn or functional Hausdorff) if for distinct p, $q \in X$ there exists a map $f \in \mathrm{C}(X)$, the set of all real valued continuous functions

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