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SENTENTIAL NOTATIONS: UNIQUE DECOMPOSITION

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Two notations for sentential logic are compared: that of Chapter II of *Logic: Techniques of Formal Reasoning* (New York, 1964) by Donald Kalish and Richard Montague and a parenthesis-free variant presented in Chapter VIII*. These notations, *SC* and *SC** respectively, are set out in section **1**, said to be unambigous in section **2**, and in sections **3** and **4** shown to be unambiguous; lastly, and briefly, in section **5** comments are made on their relative merits.

1 *The two notations* Symbols: sentence letters P through Z with or without subscripts, sentential connectives $\sim$, $\rightarrow$, $\vee$, $\wedge$, and $\leftrightarrow$, and in the case of *SC* left- and right-parentheses.

The set of sentences of *SC* is the smallest set such that: (1) Sentence letters are members of *SC*. (2) If ϕ and ψ are members of *SC*, then so are, $\sim\phi$, $(\phi \rightarrow \psi)$, $(\phi \vee \psi)$, $(\phi \wedge \psi)$, and, $(\phi \leftrightarrow \psi)$.

The set of sentences of *SC** is the smallest set such that: (1) Sentence letters are members of *SC**. (2) If ϕ and ψ are members of *SC**, then so are $\sim\phi$, $\rightarrow\phi\psi$, $\vee\phi\psi$, $\wedge\phi\psi$, and, $\leftrightarrow\phi\psi$.

The *SC*-counterpart of an *SC**-sentence is reached by successive applications of the rule:

> Where ϕ is an *SC**-sentence or sequence of *SC*-symbols and the left-most occurrence in ϕ of a binary connective is an occurrence of δ, replace the left-most occurrence in ϕ of an *SC**-sentence of the form,
>
> $$\delta\psi\chi,$$

*The lemma for Section **4** is entailed by Theorem 1 of Chapter IV, *The Elements of Mathematical Logic*, Paul Rosenbloom (New York, 1950), p. 154; see also "Bibliographical and Other Remarks," p. 205. Theorems similar to that of Section **3** are proved in *Introduction to Mathematical Logic*, Alonzo Church (Princeton, 1956), pp. 92 and 122; and in section 2 of "Proof by Cases in Formal Logic," S. C. Kleene, *Annals of Mathematics*, vol. 35 (1934), wherein can be found, see 2I, p. 531 an inductive proof for the lemma of our Section **3**. I owe these references to Alisdair Urquhart. None (I confess) were known to me before completion of this paper.

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