Notre Dame Journal of Formal Logic
Volume XX, Number 2, April 1979
NDJFAM

AN EXTENDED JOINT CONSISTENCY THEOREM FOR FREE LOGIC WITH EQUALITY

RAYMOND D. GUMB

1 *Introduction* In this paper, proof is given of an extended joint consistency theorem for free logic with equality using techniques sketched by Hintikka in a proof of a similar theorem for standard first order logic.[1] The Robinson consistency theorem and the Craig and Lyndon interpolation theorems are immediate corollaries, and proof of the Beth definability theorem is easily had. Certain related theorems of standard first order logic, though, do not hold in free logic.[2] To obtain the extended joint consistency theorem, I modify the tree method as presented by Leblanc and Wisdom for standard first order logic,[3] adapting it to free logic with equality. The completeness and soundness of a free method for free logic with equality are then demonstrated informally by showing that it is equivalent to an axiomatization known to be sound and complete. Since an extension of this tree method can be applied to any infinite set of wffs, the extension is strongly complete and strongly sound. Free logics are of interest, first, because of the insights they provide concerning a number of philosophical issues. Axiomatic versions of certain modal and tense logics have free logics as their quantificational base, and the results presented here carry over into some of these logics. Second, since free logics provide one means of formalizing partial functions, free logics are of potential use as the underlying logics of mathematical theories, and they can be applied in the analysis of algorithms. To determine the suitability of free logics for such mathematical applications, it is desirable to establish results of the sort presented here.

2 *The Leblanc tree method* The grammar and semantics of **TQC***=, a tree method for free logic with equality, are to be those of Leblanc's **QC***=, a version of free logic with equality which employs in its deductive component axiom schemas and modus ponens as the only rule of inference.[4] **QC***= is known to be complete and sound. The primitive logical constants of **QC***= are $\sim$, $\supset$, $\forall$, and $=$. For notational convenience, I shall also use $\exists$, which is defined in the usual manner. Let T, T_1, and T_2 be individual

Received November 23, 1975