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A BINARY SHEFFER OPERATOR WHICH DOES THE WORK OF QUANTIFIERS AND SENTENTIAL CONNECTIVES

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In recent years, the range of propositional systems for which binary Sheffer operators have been discovered has broadened to include various systems with multiple truth-values, modalities, and multigrade connectives (see [1] for a review). In this paper*, I present an indigenously definable binary Sheffer operator for the first order predicate calculus, and show how the technique employed there to combine quantifiers and sentential connectives in a single operator can be used to extend the previously discovered binary Sheffer operators to capture *quantified* modal systems.

We consider a stroke language containing a countable number of individual variables $x_1, x_2, \ldots$, and for each positive integer n, a countable number of n-ary predicates. If P is an n-ary predicate letter, the result of concatenating P to the left of n variable letters is a well-formed formula. If A and B are wffs, A/B is a wff. For any positive integer k and any wff A, we introduce the notation A^k (the name of a formula) as follows:

$$A^1 = A/(A/A)$$
$$\text{if } A^n = B/C, \text{ then } A^{n+1} = C/(C/C).$$

We interpret the stroke language in terms of a standard predicate language with the usual sentential connectives and quantifiers as follows. For any wffs A and B, let

$$A/B \rightleftharpoons (v) \sim (A \cdot B)$$

where $\rightleftharpoons$ indicates semantic equivalence, and v is an individual variable such that:

(i) $v = x_k$ iff $B = A^k$
(ii) v is alphabetically the first variable which does not occur in A or B iff for all k, $B \neq A^k$.

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