Notre Dame Journal of Formal Logic
Volume XX, Number 2, April 1979
NDJFAM

α-NAMING AND α-SPEEDUP THEOREMS

BARRY E. JACOBS

*Introduction** The study of computation over infinite ordinals has its roots evolving from various areas of investigation. Takeuti [33] was concerned with the problem of reducing consistency of set theory to a theory of computation on ordinal numbers. Machover [21] sought to generalize model and recursion theoretic notions to the study of infinitary languages. Jensen and Karp [11] developed a theory of primitive recursive set and ordinal functions to investigate questions of a set theoretic vein. The first author's motivation was the need of a tool for considering various levels of Gödel's constructible hierarchy [5]; the second, from lines closely related to those of Machover. From the study of definability theory and its relation to high order logics and languages evolved the work of Kreisel [14]. Later, in collaboration with Sacks (in [15]), this work blossomed into metarecursion theory, the study of computation of Church-Kleene's ω_1^{ck}. It was Kripke [16] (and independently Platek [24]) who first isolated the key notion of admissible ordinal. The study of computation over such ordinals (unifying those aforementioned cases) became known as α-recursion theory. Kripke was able to develop enough α-recursion theory to yield an infinite analogue to the Kleene T-predicate. From this he then asserted that the major results of unrelativized recursion theory (as found in Kleene [13]) held in α-recursion theory. Sacks and Simpson [27] developed relativization and consequently the priority argument in α-recursion theory. They blended recursion and model theoretic ideas in showing that the Friedberg-Muchnik solution to Post's problem generalized to α. Following this, several of Sack's students and coworkers succeeded in proving that the major results of relativized recursion theory also lift. Particularly, interesting are the works of Lerman [19], Shore [29], [30], Lerman and Simpson [20], Leggett and Shore [18]. A rather well written survey of

*The results in this paper appear in the author's Ph.D. dissertation supervised by Professor Martin D. Davis at the Courant Institute of Mathematical Sciences, New York University. This research was supported by the National Science Foundation, Grant NSF-DCR71-029039-AOB.

Received June 18, 1977