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A GENERAL PROPOSITIONAL LOGIC OF CONDITIONALS

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Many English conditionals are not truth-functional. It follows that the theory of logical consequence embodied in truth-functional formal logic has limited application to ordinary English arguments. For example, it cannot even explain the validity of modus ponens (**MP**) or the hypothetical syllogism (**HS**) when the conditionals involved are, say, subjunctive. A theory of wider application is obviously desirable. I propose here to develop a very general propositional theory of this sort.

Non-truth-functional accounts of the conditional are not of course new. Perhaps the best known are Lewis' systems of strict implication [1]. However, it would appear that Lewis' theories have even less to do with ordinary English than does truth-functional logic, for there are hardly any English 'if p, then q's that mean "'p' implies 'q'". Nonetheless a variant of this implicational account seems to me correct (perhaps in virtue of its imprecision). Someone asserting 'if p, then q' generally makes tacit appeal to some set of conditions which, together with the truth of 'p', would yield the truth of 'q'. If these tacit conditions are expressed by a set Γ of sentences and 'yield' is taken to mean 'implies', we obtain the following semantics for the conditional: 'if p, then q' is true iff 'q' is a consequence of $\Gamma \cup \{\text{'}p\text{'}\}$ and the sentences of Γ are true. In section 1 a formal propositional logic of conditionals **C** is developed from this semantics. **C**'s Gentzen-style proof apparatus is shown to be complete in section 2. Section 3 concludes the paper with some metalogical remarks.

1 *Syntax and Semantics of* **C** The symbols of **C** are

$$- \quad \vee \quad \rightarrow \quad , \quad (\quad)$$

plus an infinite decidable set of symbols distinct from these which shall be called 'sentence letters' but not further specified. An *expression* of **C** is any finite array of symbols of **C**. *Sentences* of **C** are defined through the following sequence of clauses:

1. Sentence letters are sentences of level 0.
2. If σ is a sentence of level k, $\ulcorner -\sigma \urcorner$ is a sentence of level k.

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