68
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NEGATION AS A SIGN OF NEGATIVE JUDGMENT

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1 *Introduction* We need to form negative as well as affirmative statements because we need to mark falsity as well as truth, to register rejection as false as well as acceptance as true, and to deny as well as to assert. But we do not need an embeddable negation operator any more than we need an embeddable affirmation operator, provided operators are available for forming conjunctions, disjunctions, conditionals, and universal and existential generalizations. This thesis, which is examined and defended in what follows, is of purely theoretical significance. Its significance is at most theoretical because there is nothing wrong, and much that is convenient, in having an embeddable negation operator. But it seems to me to be of some philosophical importance in relation to the question of the meaning of negation. In particular, it opens the way for an attempt to construe the meaning of negation as deriving from the mental or behavioral phenomena of negative judgment, disbelief, and denial.

These notions can be made more precise as follows. Let $\mathcal{L}$ be a first-order language with primitive operators $\wedge$, $\vee$, $\supset$, $\sim$, $\forall$, and $\exists$, employed and understood as usual. Let $\mathcal{L}_\alpha$ be just like $\mathcal{L}$, semantically as well as syntactically, except for lacking the negation operator. $\mathcal{L}_\alpha$ is thus the negationless sublanguage of $\mathcal{L}$. Now $\mathcal{L}_\alpha$ is expressively weaker than $\mathcal{L}$; that is, there are sentences of $\mathcal{L}$ to which no sentence of $\mathcal{L}_\alpha$ is logically equivalent. Moreover, it is hard to see how the logic of $\mathcal{L}_\alpha$ could be completely formalized in an "intrinsic" manner—i.e., without allowing in formal proofs or derivations excursions through sentences of $\mathcal{L}$ that involve negation, or using at least external signs representing falsity or denial. Both of these deficiencies, however, can be made up by extending $\mathcal{L}_\alpha$ just so as to permit formation of a sentence $\sim A$ for each *negation-free* sentence A, with $\sim$ understood just as in $\mathcal{L}$. Let us call the resulting language $\mathcal{L}^*$. It is easy to characterize $\mathcal{L}^*$, syntactically and semantically, as a self-contained language. At the same time, $\mathcal{L}^*$ is a sublanguage of $\mathcal{L}$.

Now $\mathcal{L}^*$ is like $\mathcal{L}$ except that negations, sentences of the form $\sim A$, never occur as proper constituents of sentences of $\mathcal{L}^*$. This is what is

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